Microwave Frequency Translator

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Abstract—A stepped phase-shift approach, employing semiconductor switching techniques in waveguide, is used to achieve frequency translation at microwave frequencies. Stepped phase shift is employed to approximate a continuous or ideal sawtooth phase shift. It has been shown by Fourier analysis that three is the minimum number of phase steps required to achieve frequency translation with suppression of the carrier and first symmetrical sideband. A tunable device using microwave switching diodes in a single port Y junction is described. The diodes progressively switch three lenths of waveguide into the circuit establishing three phase steps. A ferrite circulator is used to create a two port device and a modulator supplies proper diode biases and switching logic. Carrier suppression of greater than 30 dB and first symmetrical sideband suppression of greater than 20 dB was observed; other sideband amplitudes are predictable. A conversion efficiency of —6 dB including the circulator loss was measured and the bandwidth for 20 dB carrier suppression varies from almost one per cent to three per cent, depending on other suppression criteria.

AN IDEAL FREQUENCY translator can be defined as a device that will act upon an input signal having a certain frequency in such a way as to produce an output signal whose frequency is shifted by some desired amount from that of the input. Ideally, this will be accomplished without loss of power or without the generation of unwanted frequencies.

Frequency translation generally involves a modulation process that results in the generation of upper and lower sidebands of frequencies differing from the carrier by multiples of the modulation frequency. The goal is to achieve lossless frequency translation to the first sideband with suppression of the carrier, suppression of the first symmetrical sideband, and suppression of all other sidebands to as great an extent as possible. It is also desired to accomplish this with little expenditure of modulation power. Generally, the translation frequency is only a small percentage of the carrier frequency, although for practical purposes it is desirable to accomplish translation over as wide a range as possible.

Many devices producing frequency translation have been reported in the literature [1]–[8]; however, all fall short of the ideal in one respect or another. The simplest in concept is the continuous linear phase shifter. Fox [1] first demonstrated this in 1940. Although his was a mechanical device, it perhaps comes closest to achieving ideal frequency translation. The phase shifter consists of three sections of circular waveguide in tandem. The central section is free to rotate about the longitudinal axis. The first section consists of a quarter-wave plate which converts linear polarization to circular polarization. The central section contains a half-wave plate which reverses the sense of the circularly polarized wave. The third section contains a second quarter-wave plate to reconvert circular polarization to linear polarization. Change in phase is accomplished by rotating the central section. Continuous rotation will cause continuous advancement or retardation of phase. It can thus be seen that rotation at a fixed speed will cause a fixed increase or decrease in the frequency of the transmitted wave. The limitation on this device is an obvious one. A mechanical translator of this type is limited in translation frequency, at best, to a few hundred cycles per second.

Cacheris [3] demonstrated an electronic version of the Fox phase shifter. He applied the double refraction properties of ferrite materials to obtain a half-wave plate. Instead of rotating the half-wave plate mechanically, Cacheris was able to obtain a rotating magnetic field by means of two pairs of coils oriented at right angles to each other and excited 90 degrees out of phase. Continuous rotation of the magnetic field will thus cause a fixed increase or decrease in frequency of the transmitted signal. Frequency shifts of the order of ± 20 kc/s can be obtained with carrier suppression greater than 20 dB, and sideband suppression greater than 35 dB.

An approximation to the continuous phase shifter can be obtained by means of a sawtooth function with a fast fall time. The amplitude is controlled so that 360 degrees of phase shift is achieved during the rise of the waveform. Soohoo [4] applied sawtooth drive to a Faraday rotation type phase shifter to effect frequency translation. Continuous phase shift is not possible in this device since magnetic saturation occurs after two or three cycles. The advantage of this type over the rotating magnetic field of Cacheris lies in the smaller magnetic field requirement for Faraday rotation.

The serrodyne is a frequency translator which employs linear sawtooth modulation of transit time in a transit-time device such as a klystron or traveling-wave tube. Cummings [5] was able to produce frequency shifts in a traveling-wave tube from subaudio to 57 Mc/s with at least 20 dB suppression of undesired components. The conversion loss was less than 1 dB of the normal amplifier output.

The sawtooth principle can also be applied to Hardin's [7] electronically variable phase shifter employing varractor diodes. Here the variable capacitive properties are used to obtain phase shift. The limitation on this

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device at the present time is the small range of capacitance.

Another approach to achieving frequency translation is based on the balanced modulator principle. The design was first described in the Radiation Laboratory Series [2]. Silicon diodes were employed backed by short circuits placed a fourth-wavelength behind the diode centerline. The diodes approximate balanced modulators. When the modulating voltages are applied in phase quadrature, and the RF voltages are combined in phase quadratures then the outputs are translated in frequency. The same principle was applied by Clavin [8] using ferrite devices for the balanced modulators. The limitations on translation frequency are the same as other Faraday rotation devices (approximately 100 kc), primarily caused by the "shorted turn" effect of the waveguide wall.

The method to be discussed here employs a stepped approximation to the sawtooth. It is an extension of the method described by Rutz [6] employing diode switching.

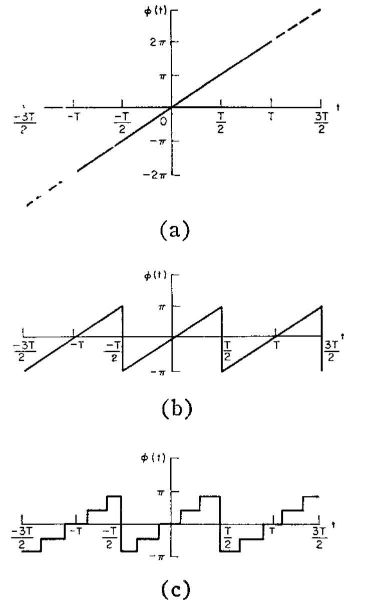

Fig. 1. Phase functions. (a) Linear continuous phase function. (b) Ideal sawtooth phase function. (c) Stepped phase function.

Semiconductor Devices

Advances in semiconductor techniques have brought about new methods of microwave frequency translation as well as a re-examination of older approaches. In prospect, the use of semiconductors may overcome most of the limitations inherent in the foregoing methods. The switching times are very fast and the drive requirements extremely small, so that large frequency offsets are possible. This should make the semiconductor devices far superior to those employing ferrite materials while the size and "passive" nature of the semiconductor diode make it more attractive than the traveling-wave tube serrodyne approach. The primary disadvantage to date has been the low power handling capabilities of the crystal diode. Recent advances [9] have resulted in diodes capable of switching tens of watts of CW microwave power at X band.

Stepped Phase Shift Approach

Figure 1 illustrates the stepped phase-shift approximation to the continuous linear phase shift required for ideal frequency translation. For a sinusoidal input waveform of unit amplitude and frequency f_0 with a phase function $\phi(t)$, the output waveform may be expressed as

$$e_0 = \sin\left[2\pi f_0 t + \phi(t)\right]. \tag{1}$$

For the case of the linear continuous phase function and the periodic ideal sawtooth of Fig. 1, $\phi(t) = 2\pi f_1 t$, where f_1 is the modulation frequency. Hence for these two cases

$$e_0 = \sin\left(2\pi f_0 t + 2\pi f_1 t\right) = \sin\left(\omega_0 + \omega_1\right)t \tag{2}$$

which is ideal frequency translation. The best stepped approximation to these ideal functions assumes equal time steps and equal phase steps. Rutz and Dye [6] have shown by Fourier analysis that three is the minimum number of phase steps required to achieve translation with suppression of the carrier and first symmetrical sideband.

The phase function for this case becomes

$$\left.\begin{array}{lll} \phi(t) = -\dfrac{2\pi}{3} & \text{for} & -\dfrac{T}{2} \leq t \leq -\dfrac{T}{6} \\ \phi(t) = 0 & \text{for} & -\dfrac{T}{6} \leq t \leq +\dfrac{T}{6} \\ \phi(t) = +\dfrac{2\pi}{3} & \text{for} & +\dfrac{T}{6} \leq t < +\dfrac{T}{2} \end{array}\right\} \tag{3}$$

where T = modulation period.

The Fourier analysis for the ideal three-step case produces spectral lines only at every third modulating frequency interval with the intervening lines going to zero. The amplitudes of the remaining lines (relative to the unmodulated carrier) can be shown to be [see Appendix a)]

$$\left.\begin{array}{ll} |E_{n+}| = \dfrac{K_0}{n} & \text{for } n = 1, 4, 7, 10 \cdots \\ |E_{n-}| = \dfrac{K_0}{n} & \text{for } n = 2, 5, 8, 11 \cdots \end{array}\right\} \tag{4}$$

where $K_0 = 3\sqrt{3}/2\pi$ and $|E_{n\pm}|$ is the magnitude of the spectral line at $(f_0 \pm n f_1)$. A plot of this spectrum is shown in Fig. 2. Since K_0 is the amplitude of the domi-

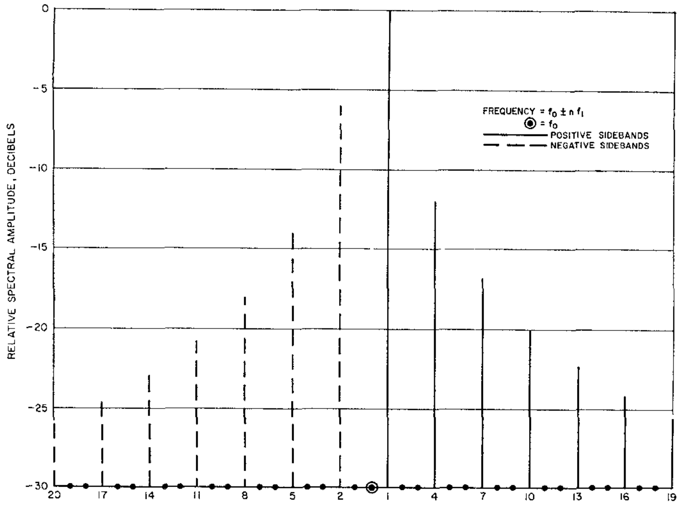

Fig. 2. Ideal spectral response—three-step phase function.

nant (or translated frequency) line, it represents the ideal conversion efficiency for the three-step frequency translator. Evaluating K_0 establishes -1.65 dB as the maximum conversion efficiency that can be expected from the translator.

The analysis of Rutz and Dye is called "ideal" since it assumes that the phase steps are exactly equal in phase and have amplitudes equal to unity. Since in actual practice this is not usually the case, it is of interest to examine the general case.

For the general case the output waveform becomes

$$e_0 = g(t) \sin [\omega_0 t + \phi(t)]$$

where $g(t)$ represents the amplitude function and $\phi(t)$ the phase function. The time division is assumed to be equal as in the ideal case. Thus for the three-step case

$$\left.\begin{array}{lll} \left.\begin{array}{l} g(t) = 1 \\ \phi(t) = \phi_1 \end{array}\right\} & \text{for } -\dfrac{T}{2} \leq t \leq -\dfrac{T}{6} \\ \left.\begin{array}{l} g(t) = A = \\ \phi(t) = \phi_2 = 0 \end{array}\right\} & \text{for } -\dfrac{T}{6} \leq t \leq +\dfrac{T}{6} \\ \left.\begin{array}{l} g(t) = B \\ \phi(t) = \phi_3 \end{array}\right\} & \text{for } +\dfrac{T}{6} \leq t \leq +\dfrac{T}{2} \end{array}\right\}. \quad (5)$$

A Fourier analysis [Appendix b)] using these values results in the following amplitudes for the spectral lines:

$$\left.\begin{array}{lll} |E_{n+}| = \dfrac{K_1}{n} & \text{for } n = 1, 4, 7, \cdots \\ |E_{n+}| = \dfrac{K_2}{n} & \text{for } n = 2, 5, 8, \cdots \\ |E_{n-}| = \dfrac{K_2}{n} & \text{for } n = 1, 4, 7, \cdots \\ |E_{n-}| = \dfrac{K_1}{n} & \text{for } n = 2, 5, 8, \cdots \end{array}\right\} \quad (6)$$

where K_1 is the conversion efficiency and K_2 is the "suppression" factor (so-called since it is a measure of the degree of suppression of the normally zero spectral lines for the ideal case). Both K_1 and K_2 are intricately related to $g(t)$ and $\phi(t)$ as is also carrier suppression. It is interesting to note that even in the general case the spectral lines that have frequency differences from the carrier equal to a multiple of three times the modulation frequency are completely suppressed.

From this analysis we can develop the following set of conclusions:

1) Suppression of spectral lines at frequencies equal to $f_0 \pm 3kf_1$ ($k = 1, 2, 3, \cdots$) is dependent on the time function only and is independent of the amplitude and phase of the stepped phase function itself.

2) The normalized amplitude of the positive sidebands at $n = 4, 7, 10 \cdots$ and negative sidebands at $2, 5, 8, 11 \cdots$ is always equal to $1/n$ and is independent of the amplitude and phase of the stepped phase function itself.

3) Conversion efficiency and carrier suppression are dependent on the amplitude and phase of the stepped phase function.

4) The suppressed amplitude of the symmetrical sideband, as well as sidebands not covered by conclusions 1) and 2), is equal to K_2/n where the suppression factor (K_2) is a function of the amplitude and phase of the stepped phase function.

Mechanization

The frequency translator is composed of three main units: a ferrite circulator; a phase modulator unit, and a modulator driver. The circulator enables a single port tunable reflective type phase modulator to become a two port component that may be inserted in series in a microwave circuit. The phase modulator is basically a shorted piece of waveguide whose length (and thus its insertion phase) can be varied by means of microwave switching diodes. The modulator driver provides the bias voltages and currents for the switching diodes in the proper time sequence. Figure 3 illustrates the device.

The modulator unit consists of a symmetrical Y junction with two germanium switching diodes placed across the narrow dimension of the waveguide and located near the juncture. Figure 4 is a photograph of the phase modulator. Two arms of the Y are terminated with adjustable short circuits. The third arm functions as a single input and output port. The diodes are arranged so that when both are reverse biased (Condition I) the microwave energy entering the input port is reflected from the plane of the diodes. When diode 2 is reflecting and diode

1 passing (Condition II), energy passes into arm 1 and is reflected from adjustable short 1 back to the input. When diode 1 is reflecting and diode 2 passing (Condition III), energy is reflected from adjustable short 2.

Condition I is the reference phase condition. Short 1 is adjusted so that under Condition II the effective length of arm 1 is one-sixth of a guide wavelength. The two-way length is one-third of a wavelength which adds 120 degrees of phase shift relative to Condition I. Short 2 is adjusted so that under Condition III the effective length of arm 2 is one-third of a wavelength, adding 240 degrees relative to Condition I.

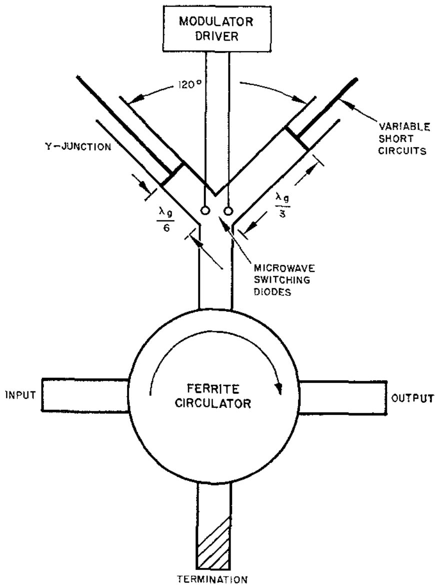

Fig. 3. Schematic arrangement—stepped phase-shift frequency translator.

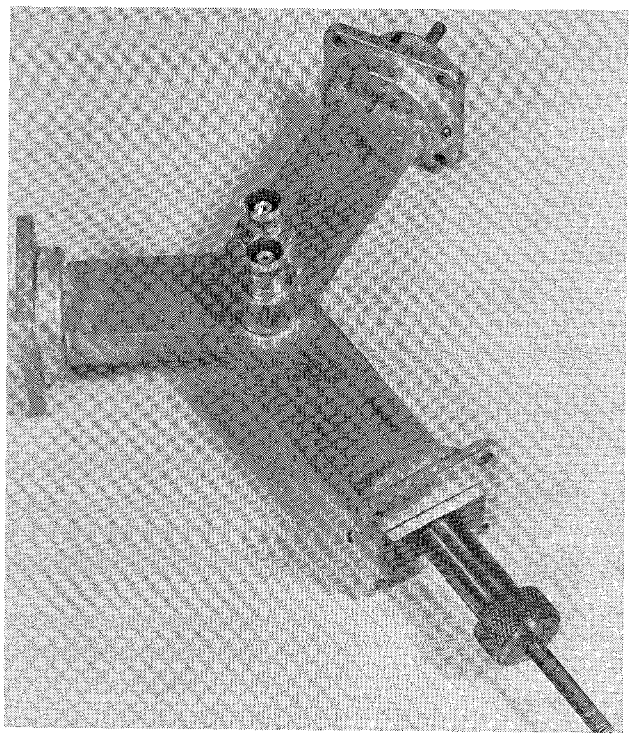

Fig. 4. The phase modulator unit.

The diodes are placed so that they lie on the centerlines of Arms 1 and 2, respectively. The position of the diodes along the centerline was determined by treating the input guide and each side arm as a 120 degree mitred corner waveguide bend. Diode 2 is considered as the corner plate on bend 1 and diode 1 the corner plate for bend 2. Measurements established that the plane of reflection does indeed lie very nearly along the diode axis.

Switching Diodes

The important characteristics of microwave switching diodes for this application are:

1) Minimum insertion loss in the forward bias condition
2) Maximum reflectivity in the reverse bias condition
3) Fast switching time
4) Capability of handling appreciable RF power.

The diode equivalent circuit and mode of operation are well known [10], [11] and will not be repeated here. The diodes used were Philco types 1N3481, 1N3482, and 1N3039. The primary difference between them is power handling capability. The 1N3481 is rated at only 1 milliwatt, whereas the 1N3093 and 1N3482 are rated at 0.5 and 1.25 watts, respectively. The 1N3093 was selected for demonstrating the frequency translator, representing a compromise between power handling capability and drive requirements. Typical numbers for this diode are:

1) Forward insertion loss (at 60 mA) 1.5 dB
2) Reverse bias isolation (11 V) 21 dB
3) Switching time $\sim 10^{-9}$ sec.

Modulator Driver

In describing the phase modulator unit the conditions were set forth for attaining successively the three phase steps required for frequency translation. Figure 5 shows the required waveforms for each of the diodes; ON refers to the diode in forward bias or "passing" condition and OFF refers to the reverse bias or "reflecting" condition. A modulation frequency of 100 kc was chosen to demonstrate feasibility although the analysis indicates capability at considerably higher frequencies. Translation is presently limited by the driver rather than by the microwave diodes. Current state of the art of transistor multivibrators and gating circuitry indicates that frequency translations up to 100 Mc should be possible.

A block diagram of the modulator is shown in Figure 6. A 300 kc clock rate is used as events occur at one-third cycle intervals. Three time staggered gates are required each passing every third pulse. The output from AND gate 1 permits diode 1 to be turned ON and initiates gate generator 2. The output from AND gate 2 turns diode 1 OFF, diode 2 ON and initiates gate generator 3. The output from AND gate 3 turns diode 2 OFF and initiates gate generator 1. In theory, a starting pulse is required to initiate gate generator 1 after which the process is self-sustaining; in practice the turn-on transient was sufficient to trigger the process.

The transistor output amplifiers provide a pulse of current to the 1N3093 diodes of sufficient amplitude to bias them ON. A reverse bias supply keeps the diodes in the OFF state between current pulses. From detailed measurements of insertion loss in the forward and reverse bias conditions, on ON current of 70 mA was optimum for both diodes while the optimum reverse bias was 7.5 volts for diode 1 and 5.0 volts for diode 2. The modulator rise and fall times are 0.1 and 0.2μs, respectively.

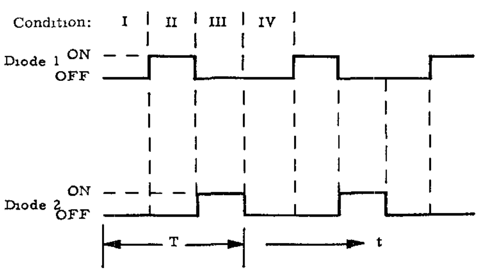

Fig. 5. Diode waveforms.

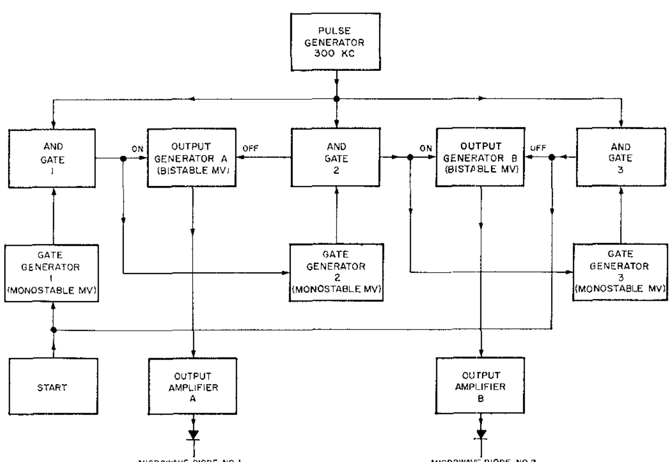

Fig. 6. Modulator driver block diagram.

Experimental Results

Loss measurements were made on the combined circulator and Y junction with the following results.

Condition	Diode 1	Diode 2	Insertion loss (dB) through device
I	OFF	OFF	2.5
II	ON	OFF	5.5
III	OFF	ON	5.7

These results are in close agreement with measurements made separately on the circulator and phase modulator. It is obvious that this mechanization cannot result in equal amplitude signals for all phase steps since in Condition I the two-way insertion loss of the forward biased diodes is not present.

Measurement of the spectral output of the translator was made after tuning the phase shifter by three different methods. The first method (Case I) consisted of carefully adjusting the positions of the shorts to $\lambda g/6$ and $\lambda g/3$ by measuring relative minimum positions of the standing wave using a slotted section and referenc- the zero phase to the condition of reverse bias on both diodes. The second method (Case II) consisted of adjusting the short positions for maximum suppression of the carrier as observed on a spectrum analyzer. Case III was like Case II with the exception that a slide-screw tuner was placed between the circulator and Y junction. Both short positions and tuner were adjusted for maximum carrier rejection. Case IV, a variation of Case III, was an adjustment for equal maximum suppression of carrier and first symmetrical sideband. The carrier frequency was 9 Gc and amplitude measurements were made relative to the dominant spectral line. Figure 7

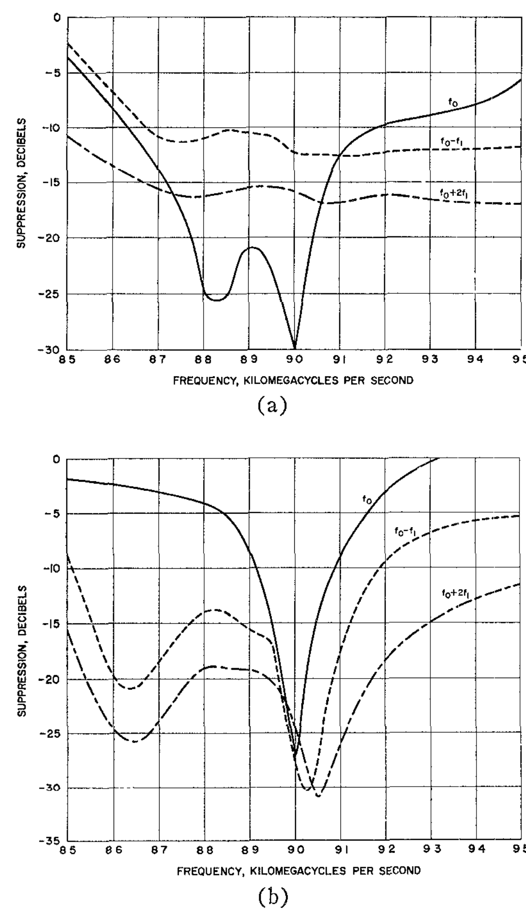

Fig. 7. Suppression vs. microwave frequency. (a) Case II: Phase adjusted for maximum carrier suppression at 9.0 Gc. (b) Case IV: Phase and impedance adjusted for equal carrier and sideband suppression at 9.0 Gc.

shows that the improved suppression of Case IV is achieved at the expense of bandwidth over Case II.

Using the relationships presented earlier and derived in the Appendix, it is possible to calculate the efficiency and suppression factors K_1 and K_2, respectively, for three of these cases. (Case IV was a minor change from Case III.) This was done using measured values of amplitude and phase. The suppressed carrier amplitude (E_0) can also be calculated, as shown in Table I.

Using these factors and a table of $1/n$, a comparison of calculated and experimental values can be made.

Conversion Efficiency

Conversion efficiency is the ratio of the power of f_0+f_1 under modulation to the power of f_0 with modulation off (diodes in Condition I). An additional fixed loss of 2.5 dB occurs in the system due to the circulator and the diodes. A comparison of calculated and measured values is shown in Table II.

Spectral Comparison

Table III presents the calculated and measured values for a number of the sidebands for each case. All values

TABLE I

Measured and Calculated Parameter Values

Case	* A	* B	* ϕ_1	* ϕ_3	** K_1^{dB}	** K_2^{dB}	** E_0^{dB}
I	0.700	0.741	−120°	+120°	−3.4	−22.2	−20.6
II	0.630	0.727	−139°	+92°	−3.9	−15.3	−25.2
III	0.814	0.870	−135°	+102°	−2.7	−19.1	−25.3

* Measured.
** Calculated.

TABLE II

Conversion Efficiency

Case	Calculated (dB)	Measured (dB)
Ideal	−1.65	—
I	−3.4	−3.8
II	−3.9	−3.9
III	−2.7	−3.0

TABLE III

Comparison of Calculated and Measured Spectral Amplitudes

n	Positive Sidebands Relative Amplitudes (dB)							Negative Sidebands Relative Amplitudes (dB)						
Case:	Ideal	I*	I**	II*	II**	III*	III**	Ideal	I*	I**	II*	II**	III*	III**
0	−∞	−17.1	−17.4	−21.3	−30.0	−22.6	−31.8							
1	0	0	0	0	0	0	0	−∞	−18.8	−18.7	−11.4	−12.3	−16.4	−20.8
2	−∞	−24.8	−21.8	−17.4	−15.9	−22.4	−22.1	−6.0	−6.0	−6.0	−6.0	−6.0	−6.0	−6.1
3	−∞	−∞	> −40	−∞	> −40	−∞	> −30	−∞	−∞	−31.5	−∞	−30.5	−∞	> −30
4	−12.0	−12.0	−12.3	−12.0	−12.3	−12.0	−12.5	−∞	−30.8	−31.5	−23.5	−26.7	−28.4	> −30
5	−∞	−32.7	−28.1	−25.4	−23.8	−30.4	> −30	−14.0	−14.0	−14.0	−14.0	−14.1	−14.0	−14.1
6	−∞	−∞	> −40	−∞	> −40	−∞	> −30	−∞	−∞	−34.5	−∞	−33.4	−∞	> −30
7	−16.9	−16.9	−17.5	−16.9	−17.5	−16.9	−17.7	−∞	−35.7	−34.5	−28.3	−33.4	−33.3	> −30
8	−∞	−36.8	−31.3	−29.5	−28.5	−34.4	> −30	−18.1	−18.1	−18.3	−18.1	−18.1	−18.1	−18.1
9	−∞	−∞	> −40	−∞	> −40	−∞	> −30	−∞	−∞	−35	−∞	−33.6	−∞	> −30
10	−20.0	−20.0	−21.0	−20.0	−21.0	−20.0	−21.3	−∞	−38.8	−35	−31.4	−39.5	−36.4	> −30
11	−∞	−39.6	−32.5	−32.3	−31.2	−37.2	> −30	−20.8	−20.8	−24.7	−20.8	−21.2	−20.8	−21.2
12	−∞	−∞	−32.5	−∞	> −40	−∞	> −30	−∞	−∞	−35	−∞	−35.2	−∞	> −30
13	−22.3	−22.3	−23.7	−22.3	−23.3	−22.3	−23.8	−∞	−41.0	−35	−33.7	−43.3	−38.7	> −30
14	−∞	−41.7	−34.2	−34.4	−33.0	−39.3	> −30	−22.9	−22.9	−25.8	−22.9	−23 6	−22.9	−23.6
15	−∞	−∞	−34.2	−∞	−39.0	−∞	> −30	−∞	−∞	> −35	−∞	> −40	−∞	> −30
16	−24.1	−24.1	−25.8	−24.1	−25.5	−24.1	−26.2	−∞	> −40	> −35	−35.5	> −40	> −40	> −30

* Calculated.
** Measured.

are shown relative to the amplitude of the dominant spectral line at (f_0+f_1).

Conclusion

It has been demonstrated that the stepped phase shift approach meets to some degree most of the criteria established for the ideal microwave frequency translator. Translation to the first sideband with suppression of the carrier and symmetrical sideband can be realized. Other sidebands are present but are readily predictable and are several modulation intervals removed from the dominant line. Total conversion efficiencies (including circulator) of approximately -6 dB can be obtained. Translation frequencies of several megacycles appear possible with small expenditure of modulation power.

Appendix

Fourier Analysis of Stepped Phase Shift

For the general case the output waveform may be expressed as

$$e_0 = g(t) \sin [\omega_0 t + \phi(t)]$$

$$e_0 = \sin \omega_0 t[g(t) \cos \phi(t)] + \cos \omega_0 t[g(t) \sin \phi(t)]. \quad (7)$$

Representing the periodic functions as Fourier series:

$$g(t) \sin \phi(t) = a_0/2 + \sum_{n=1}^{\infty} (a_n \cos n\omega_1 t + b_n \sin n\omega_1 t)$$

$$g(t) \cos \phi(t) = c_0/2 + \sum_{n=1}^{\infty} (c_n \cos n\omega_1 t + d_n \sin n\omega_1 t)$$

$$e_0 = a_0/2 \cos \omega_0 t + c_0/2 \sin \omega_0 t$$

$$+ \frac{1}{2} \sum_{n=1}^{\infty} \left\{ \begin{array}{l} (a_n - d_n) \cos (\omega_0 + n\omega_1)t \\ + (c_n + b_n) \sin (\omega_0 + n\omega_1)t \\ + (a_n + d_n) \cos (\omega_0 - n\omega_1)t \\ + (c_n - b_n) \sin (\omega_0 - n\omega_1)t \end{array} \right\} \quad (8)$$

where

$$a_0 = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \sin \phi(t) dt$$

$$c_0 = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \cos \phi(t) dt$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \sin \phi(t) \cos n\omega_1 t dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \sin \phi(t) \sin n\omega_1 t dt$$

$$c_n = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \cos \phi(t) \cos n\omega_1 t dt$$

$$d_n = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \cos \phi(t) \sin n\omega_1 t dt.$$

a) *Ideal Case*

For the ideal three-step case, $g(t)=1$ for all values of t, and $\phi(t)$ is an odd function. Sin $\phi(t)$ is therefore an odd function but cos $\phi(t)$ is even.

Hence

$$a_n = 0 \qquad (\text{also } a_0 = 0)$$

and

$$d_n = 0$$

and it can be shown that $c_0=0$. Equation (8) now becomes

$$e_0 = 1/2 \sum_{n=1}^{\infty} [(c_n + b_n) \sin (\omega_0 + n\omega_1)t + (c_n - b_n) \sin (\omega_0 - n\omega_1)t] \quad (9)$$

c_n and b_n can be evaluated using the phase function of (3).

$$c_n + b_n = \frac{\sqrt{3}}{\pi n}\left[\sqrt{3} \sin \frac{\pi n}{3} + \cos \frac{\pi n}{3} \pm 1\right] \begin{cases} \text{use } +1 & \text{for } n \text{ odd} \\ \text{use } -1 & \text{for } n \text{ even} \end{cases}$$

$$c_n - b_n = \frac{\sqrt{3}}{n\pi}\left[\sqrt{3} \sin \frac{\pi n}{3} - \cos \frac{\pi n}{3} \pm 1\right] \begin{cases} \text{use } -1 & \text{for } n \text{ odd} \\ \text{use } +1 & \text{for } n \text{ even} \end{cases}$$

$$c_n + b_n = c_n - b_n = 0 \quad \text{for } n = 3k \ (k = 1, 2, 3, \cdots)$$

$$c_n + b_n = 0, \quad \text{for } n = 2, 5, 8, \cdots$$

$$c_n - b_n = 0, \quad \text{for } n = 1, 4, 7, \cdots$$

From (9) the magnitudes of the spectral lines are

$$|E_{n+}| = |1/2(c_n + b_n)| \quad \text{and}$$

$$|E_{n-}| = |1/2(c_n - b_n)|.$$

Then

$$|E_{n+}| = \frac{3\sqrt{3}}{2\pi n} \qquad \text{for } n = 1, 4, 7, 10, \cdots$$

and

$$|E_{n-}| = \frac{3\sqrt{3}}{2\pi n} \qquad \text{for } n = 2, 5, 8, 11, \cdots$$

b) *General case*

For the general case the phase and amplitude function can be represented by (5).

The amplitude reference is chosen as Condition I (both diodes reflecting) since the conversion efficiency is measured with reference to this condition. The phase reference is arbitrarily chosen as ϕ_2 for ease of analysis. It is now necessary to determine all of the coefficients of (8).

$$\frac{a_0}{2} = \frac{1}{3}\sin\phi_1 + \frac{B}{3}\sin\phi_3$$

$$\frac{c_0}{2} = \frac{1}{3}\cos\phi_1 + \frac{A}{3} + \frac{B}{3}\cos\phi_3$$

$$a_n = \frac{W}{\pi n}\sin\frac{\pi n}{3}$$

$$b_n = \frac{-X}{\pi n}\left(\cos\frac{\pi n}{3} - \cos\pi n\right)$$

$$c_n = \frac{-Y}{\pi n}\sin\frac{\pi n}{3}$$

$$d_n = \frac{-Z}{\pi n}\left(\cos\frac{\pi n}{3} - \cos\pi n\right)$$

where

$$W = \sin\phi_1 + B\sin\phi_3$$

$$X = \sin\phi_1 - B\sin\phi_3$$

$$Y = \cos\phi_1 - 2A + B\cos\phi_3$$

$$Z = \cos\phi_1 - B\cos\phi_3$$

$$a_n = b_n = c_n = d_n = 0 \qquad \text{for } n = 3k \ (k = 1, 2, 3, \cdots)$$

as in the ideal case.

From (8) the magnitude of the suppressed carrier is

$$|E_0| = \sqrt{\left(\frac{a_0}{2}\right)^2 + \left(\frac{c_0}{2}\right)^2}$$

and the sideband magnitudes are

$$|E_{n+}| = \sqrt{\left(\frac{a_n - d_n}{2}\right)^2 + \left(\frac{c_n + b_n}{2}\right)^2}$$

$$|E_{n-}| = \sqrt{\left(\frac{a_n + d_n}{2}\right)^2 + \left(\frac{c_n - b_n}{2}\right)^2}.$$

Substituting the coefficients

$$|E_{n+}| = \frac{K_1}{n} \qquad \text{for } n = 1, 4, 7, \cdots$$

$$|E_{n+}| = \frac{K_2}{n} \qquad \text{for } n = 2, 5, 8, \cdots$$

$$|E_{n-}| = \frac{K_2}{n} \qquad \text{for } n = 1, 4, 7, \cdots$$

$$|E_{n-}| = \frac{K_1}{n} \qquad \text{for } n = 2, 5, 8, \cdots$$

where

$$K_1 = \frac{1}{4\pi}\sqrt{(\sqrt{3}\,W + 3Z)^2 + (\sqrt{3}\,Y + 3X)^2}$$

$$K_2 = \frac{1}{4\pi}\sqrt{(\sqrt{3}\,W - 3Z)^2 + (\sqrt{3}\,Y - 3X)^2}.$$

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